INVESTIGATION OF THE EFFECT OF TRIPLE RECOMBINATION OF ELECTRONS AND IONS ON ELECTRODYNAMIC PLASMA ACCELERATION

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UDC 533.9.07

The effect of processes of three-particle electron–ion recombination in plasma during its electrodynamic acceleration in a coaxial channel was investigated. The appropriate systems of differential equations were solved on a computer. The integral characteristics of plasma were determined both with consideration of triple recombination of electrons and ions and with consideration of other processes of mass transfer and resistance forces.

Volume recombination of negative ions and electrons with positive ions is one of the common mechanisms of loss of charged particles from plasma [1-8]. According to the type of recombining particles, recombination of ions and electrons are distinguished; according to the number ofinteracting particles, two-particle, three-particle, etc., are distinguished. Two-particle ion recombination is possible due to the thermal motion of ions or under the effect of the attractive forces of their electrostatic field. As a rule the process of recombination occurs in the presence of a third particle, and the energy released during recombination is distributed between the three particles. It can change to the kinetic energy of the particles or to the excitation or dissociation energy of molecules. In the case of "spontaneous" recombination (which occurs in the absence of a third particle) the two particles formed carry away the excess energy in the form of excitation energy. In the case of electron recombination, one distinguishes radiative recombination, when the recombination energy is emitted as quantum, dissociative recombination, etc.

Triple recombination was accomplished as a result of triple collisions of electrons and ions with free plasma particles, such as an ion, electron, or neutral molecule.

The equation of the kinetics of three-particle recombination, according to [3], can be written in the form

$$\frac{dN_e}{dt} = -\alpha_{3\,\mathrm{recomb}} N_e N_i N. \tag{1}$$

The values of the coefficient of triple recombination $\alpha_{3\mathrm{recomb}}$, m^6/sec, can vary in a very wide range 10^{-30}-10^{-45} m^6/sec^{-1} [1, 3] depending on the plasma temperature, concentration of ions N_i and electrons N_e, concentration of neutral molecules N, pressure, and other external conditions.

If the electron and ion concentrations are about the same, which is usually true for plasma, and recombination occurs upon collision of three particles – electron, positive ion, and electron, then Eq. (1) can be transformed to

$$\frac{dN_e}{dt} = -\alpha'_{3\,\mathrm{recomb}} N_e^3, \tag{2}$$

as was done in [5].

The coefficient of three-particle recombination depends on the pressure. The appropriate expressions for the coefficients of three-particle recombination obtained by Thomson [3, 5] for low gas pressures and by Langevin [3, 5] for higher pressures (of the order of atmospheric and higher) are given in [1-8].

Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 20, No. 3, pp. 467-475, March, 1971. Original article submitted February 18, 1970.

The coefficient of three-particle recombination for molecular ions is several orders greater than that for atomic ions [5], since the portion of energy lost by the electron on collision with a molecule is several orders of magnitude greater than the portion of energy lost on collision with an atom.

The coefficient of three-particle recombination depends considerably on the electron temperature and increases rapidly with its decrease. Therefore, at low temperatures, when the ions formed in the plasma are molecular, triple recombination can compete with two-particle, since the coefficient of two-particle recombination of electrons and molecular ions increases more weakly with decrease of temperature ($\alpha_{2\text{recomb}} \sim T^{-3/2}$). This circumstance is of practical value in electron–ion recombination, for example, in helium plasma [9, 11].

The effect of loss of charged particles owing to two-particle ion recombination in plasma on the process of its electrodynamic acceleration was investigated in [12]. It is of interest to investigate the effect of three-particle recombination of electrons and ions on electrodynamic plasma acceleration. In so doing we will consider also the combined effect of two-particle ion and three-particle electron–ion recombination of plasma during its acceleration, since in many cases both mechanisms of loss of charged particles act simultaneously.

If we neglect the effect of other factors and take into account only triple recombination of ions and electrons, the equations describing the process of electrodynamic acceleration in an integral form in a coaxial channel can be written so:

$$\frac{dmv}{dt} = \frac{b}{2} I^2, \tag{3}$$

$$\frac{dz}{dt} = v, \tag{4}$$

$$\frac{dV}{dt} = -\frac{1}{C_0} I, \tag{5}$$

$$\frac{dLI}{dt} + R_0 I - V = 0, \tag{6}$$

$$L = L_0 + bz, \tag{7}$$

$$\frac{dm}{dt} = -a_5 m^3, \tag{8}$$

where Eq. (3) describes the motion of the center of mass of the accelerated plasma under the effect of magnetic forces; Eq. (4) determines the velocity of the center of mass of the plasma; Eqs. (5), (6) are the equations of the current in the circuit and voltage balance, respectively; Eq. (7) describes the law of variation of the inductance of the coaxial channel during electrodynamic plasma acceleration; and Eq. (8) is the mass balance with consideration only of three-particle recombination of plasma.

Since during electrodynamic acceleration mass transfer is accomplished mainly by ions ($m_i \gg m_e$), when $N_i \simeq N_e$ loss of ions will play the main role, naturally, in the decrease of the accelerated mass. If we introduce the plasma mass $m = m_i N_i$ as the product of the ion mass m_i and ion concentration N_i, the mass coefficient of triple recombination a_5, according to (2) and (8), will be determined by the relation

$$a_5 = \frac{\alpha_{3\,\text{recomb}}}{m_i^2}, \quad \text{m}^6 \cdot \text{sec}^{-1}/\text{kg}^2. \tag{9}$$

To solve system of Eqs. (3)-(8) we will assign the initial conditions when $t = 0$ in the form

$$m = m_0, \quad V = V_0, \quad I = 0, \quad v = 0, \quad z = 0. \tag{10}$$

Equations (3)-(8) are conveniently considered in dimensionless variables

$$\mu = \frac{m}{m_0}, \quad \varphi = \frac{V}{V_0}, \quad \varphi' = \sqrt{\frac{L_0}{C_0}} \cdot \frac{I}{V_0}, \qquad \tau = \frac{t}{\sqrt{L_0 C_0}}, \quad y = \frac{b}{L_0} z, \quad y' = b\sqrt{\frac{C_0}{L_0}}\, v. \tag{11}$$

In these variables system of Eqs. (3)-(8) can be reduced in a dimensionless form to the following canonical form:

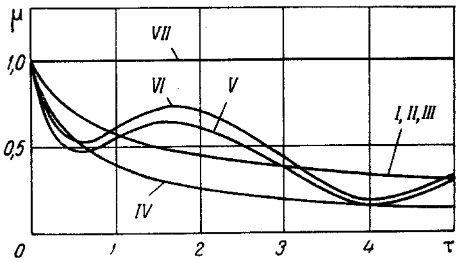

Fig. 1

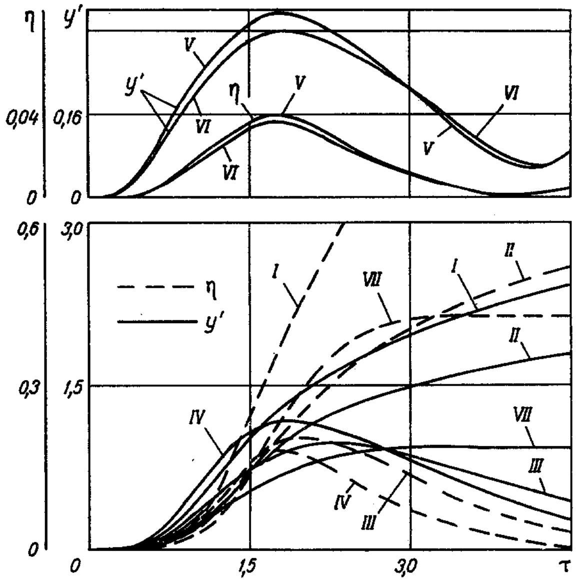

Fig. 2

Fig. 1. Change in time τ of the accelerated mass μ for the following parameters of system of Eqs. (29)-(34): q = 1, I) $\alpha = 0.1$, $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = \gamma_6 = 0$, $\gamma_5 = 1$, $\delta_1 = \delta_2 = \delta_3 = \delta_4 = 0$; II) $\alpha = 0.5$, $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = \gamma_6 = 0$, $\gamma_5 = 1$, $\delta_1 = \delta_2 = \delta_3 = \delta_4 = 0$; III) $\alpha = 0.1$, $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = \gamma_6 = 0$, $\gamma_5 = 1$, $\delta_1 = \delta_2 = \delta_3 = \delta_4 = 0.1$; IV) $\alpha = 0.1$, $\gamma_1 = \gamma_3 = \gamma_4 = \gamma_6 = 0$, $\gamma_2 = \gamma_5 = 1$, $\delta_1 = \delta_2 = \delta_3 = \delta_4 = 0.1$; V) $\alpha = 0.1$, $\gamma_6 - \gamma_1 = 1$, $\gamma_2 = \gamma_3 = \gamma_4 = \gamma_5 = 1$, $\delta_1 = \delta_2 = \delta_3 = \delta_4 = 0.1$; VI) $\alpha = 0.1$, $\gamma_6 - \gamma_1 = 1$, $\gamma_2 = \gamma_3 = \gamma_4 = 1$, $\gamma_5 = 0.1$, $\delta_1 = \delta_2 = \delta_3 = \delta_4 = 0.1$; VII) $\alpha = 0.1$, $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = \gamma_5 = \gamma_6 = \delta_1 = \delta_2 = \delta_3 = \delta_4 = 0$.

Fig. 2. Change of center-of-mass velocity y' and efficiency η in time τ. The parameters and notations correspond to Fig. 1.

$$\frac{dy'}{d\tau} = \frac{q\varphi'^2}{\mu} + \gamma_5 y' \mu^2, \tag{12}$$

$$\frac{dy}{d\tau} = y', \tag{13}$$

$$\frac{d\varphi}{d\tau} = -\varphi', \tag{14}$$

$$\frac{d\varphi'}{d\tau} = -\frac{(\alpha + y')\,\varphi'}{1+y} + \frac{\varphi}{1+y}, \tag{15}$$

$$\frac{d\mu}{d\tau} = -\gamma_5\,\mu^3, \tag{16}$$

where

$$q = \frac{b^2 C_0^2 V_0^2}{2 m_0 L_0}, \quad \alpha = R_0 \sqrt{\frac{C_0}{L_0}}, \quad \gamma_5 = a_5 m_0^2 \sqrt{L_0 C_0}. \tag{17}$$

The initial conditions (10) in variables (12) will take on the form

$$\varphi = \mu = 1, \quad y' = y = \varphi' \text{ when } \tau = 0. \tag{18}$$

The system of equations obtained is characterized by three dimensionless parameters (17): q, α, and γ_5. The parameter q represents the energy parameter of Artsimovich et al. [17], and the parameter α characterizes the dimensionless ohmic resistance of the circuit. Their physical sense and values have already been discussed in [12-17].

The parameter γ_5 in relative values characterizes the change (loss) of mass due to triple recombination of electrons and ions in plasma. Depending on the character of electrodynamic acceleration and parameters of the accelerator (m_0, C_0, L_0, a_5) the value of γ_5 can vary within a wide range from 0 to 10^2 and more.

Thus, for $L_0 = 50 \cdot 10^{-9}$ H, $C_0 = 100 \cdot 10^{-6}$ F, $\alpha'_{3\text{recomb}} \simeq 10^{-35}$ m^6/sec, $m_0 = 10^{-6}$ kg, and $m_i = 3 \cdot 10^{-27}$ kg, the quantity γ_5 is determined, according to (18), in the following way:

$$\gamma_5 = \frac{\alpha'_{3\,\text{recomb}}\, m_0^2 \sqrt{L_0 C_0}}{m_i^2} = \frac{10^{-35} \cdot 10^{-6} \cdot 10^{-6} \cdot 10^{-6}}{3 \cdot 10^{-27} \cdot 3 \cdot 10^{-27}} \sim 1. \quad (19)$$

System of Eqs. (12)-(16) was integrated numerically for initial conditions (18). The results of the solution are presented in Figs. 1-3. Curves I of these figures permit evaluating the effect of triple recombination of ions and electrons for parameters of system (17) ($q = 1$, $\alpha = 0.1$, and $\gamma_5 = 1$) on the integral characteristics of the accelerated plasma – velocity of the center of mass y'; voltage φ and current φ' in the accelerator; change in time τ of the accelerated mass μ; and efficiency η. The efficiency η was determined as [15, 16]

$$\eta = \mu y'^2 / 2q. \quad (20)$$

Curves II permit evaluating the effect on the indicated characteristics of a fivefold increase of resistance α in comparison with curves I.

Curves VII are the solution of the system of equations describing electrodynamic plasma acceleration on the basis of an idealized current connector [17] when mass release during acceleration is absent:

$$\frac{dy'}{d\tau} = q\varphi'^2, \quad (21)$$

$$\frac{dy}{d\tau} = y', \quad (22)$$

$$\frac{d\varphi}{d\tau} = -\varphi', \quad (23)$$

$$\frac{d\varphi'}{d\tau} = \frac{\varphi - (\alpha + y')\varphi'}{1 + y} \quad (24)$$

for initial conditions

$$\varphi = 1, \ \varphi' = y' = y = 0 \quad \text{when} \quad \tau = 0. \quad (25)$$

It follows from Fig. 1 that triple recombination of electrons and ions leads to a decrease of the mass of the accelerated plasma μ. Thus, by time $\tau \approx 1.5$ it decreases by half and has the value $\mu = 0.5$. Its decrease leads to a rapid increase of the velocity of the plasma y' and efficiency in time τ (Fig. 2). The effective values of the current φ' and voltage φ decrease also in this case (Fig. 3) in comparison with curves VII of these figures.

Consideration in the equation of motion (12) of the resistance due to friction of the moving plasma on the electrodes, friction during mass transfer, and resistance of the external environment requires its change, and according to [13, 14] it can be written in a dimensionless form as

$$\frac{dy'}{d\tau} = \frac{q}{\mu}\varphi'^2 - \frac{y'}{\mu}(\delta_1 + \delta_2\mu + \delta_3|\varphi'| + \delta_4 y') + \gamma_5 y'\mu^2, \quad (26)$$

where

$$\delta_1 = \frac{b_1}{m_0}\sqrt{L_0 C_0}, \quad \delta_2 = b_2\sqrt{L_0 C_0},$$
$$\delta_3 = \frac{1}{2} \cdot \frac{m_i}{e} \cdot \frac{C_0 V_0}{m_0}, \quad \delta_4 = \frac{b_4 L_0}{m_0 b}. \quad (27)$$

The values of b_1, b_2, b_3, b_4 and respectively $\delta_1 - \delta_4$ have already been discussed in [13, 14].

A comparison of Eqs. (12), (21), and (26) permits noting the appearance of an additional "reactive" force

$$\bar{F}_{\text{tr. recomb}} = \gamma_5 y'\mu^2,$$

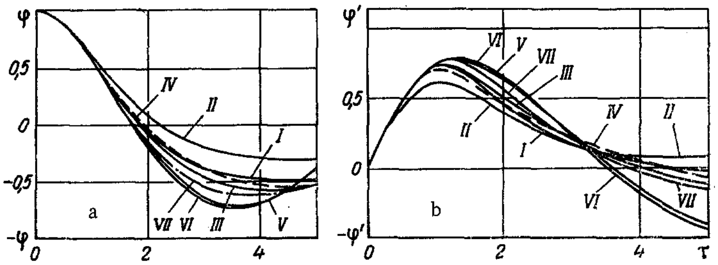

Fig. 3. Change in time τ of the voltage φ(a) and current φ' (b). The parameters and notations correspond to Fig. 1.

which appeared as a result of a decrease of mass due to three-particle electron–ion recombination. According to Eq. (26), the force $\overline{F}_{tr.recomb}$ promotes acceleration of plasma and reduces the total effect of other resistance forces considered, which are inevitable during acceleration.

Curves III are the solution of system of Eqs. (26), (13)-(16) for initial conditions (18), where $\gamma_1 = 1$ and $\delta_1 = \delta_2 = \delta_3 = \delta_4 = 0.1$. Consideration of the resistance forces leads to a marked decrease of the plasma velocity y' and efficiency η. A clear-cut maximum of their change appears, which is reached by time $\tau \simeq 1.6$-1.7.

In real accelerators two- and three-particle recombination of plasma often act simultaneously. Therefore, we will consider their combined effect on acceleration.

System of Eqs. (12)-(16) with consideration of two-particle ion recombination of the plasma can be rewritten in a dimensionless form so:

a) equation of motion

$$\frac{dy'}{d\tau} = \frac{q}{\mu}\varphi'^2 - \frac{y'}{\mu}(\delta_1 + \delta_2\mu + \delta_3|\varphi'| + \delta_4 y') + \gamma_2\mu y' + \gamma_5 y'\mu^2; \tag{28}$$

b) equation of mass balance with consideration only of the indicated mass transfer processes

$$\frac{d\mu}{d\tau} = -\gamma_2\mu^2 - \gamma_5\mu^3; \tag{29}$$

c) equations which are the determinations of the path and current and equations of the voltage balance in the circuit, which coincide with (12)-(14).

The dimensionless parameters

$$\gamma_2 = a_2 m_0 \sqrt{L_0 C_0}, \quad a_2 = \frac{\alpha_2}{m_i}, \tag{30}$$

are introduced in Eqs. (28), (29) according to [12].

An analysis of curves IV, which are the solution of Eqs. (28), (29), (12)-(14) with initial conditions (18) for the selected parameters shows that two-particle recombination in the case being considered leads to a more intense decrease of mass during acceleration.

In [14] the authors investigated the combined effect on electrodynamic plasma acceleration of mass transfer processes caused by diffusion and two-particle recombination of particles of the accelerated plasma and by an increase of its mass owing to partial melting of the electrodes and sticking of electrons and ions, and studied the effect of resistance and friction forces on acceleration. Generalizing Eqs. (12)-(16) of [14] to the case of consideration of the effect on electrodynamic acceleration of three-particle electron–ion recombination of plasma and ionization of neutral undisturbed gas, we can obtain the following equations of motion and balance of the mass in a dimensionless form:

$$\frac{dy'}{d\tau} = \frac{q}{\mu}\varphi'^2 - \frac{y'}{\mu}(\delta_1 + \delta_2\mu + \delta_3|\varphi'| + \delta_4 y') - \frac{y'}{\mu}(-\gamma_1\mu - \gamma_2\mu^2 + \gamma_3|\varphi'| + \gamma_4\varphi'^2 - \gamma_5\mu^3 + \gamma_6\mu), \tag{31}$$

$$\frac{d\mu}{d\tau} = -\gamma_1\mu - \gamma_2\mu^2 + \gamma_3|\varphi'| + \gamma_4\varphi'^2 - \gamma_5\mu^3 + \gamma_6\mu, \tag{32}$$

where

$$\gamma_1 = a_1 \sqrt{L_0 C_0}, \quad \gamma_2 = a_2 m_0 \sqrt{L_0 C_0},$$

$$\gamma_3 = \frac{a_3 C_0 V_0}{m_0}, \quad \gamma_4 = \frac{C_0^2 V_0^2}{\sqrt{L_0 C_0}} \cdot \frac{a_4}{m_0}, \tag{33}$$

$$\gamma_6 = a_6 \sqrt{L_0 C_0}.$$

The values of the parameters γ_1, γ_2, γ_3, γ_4, γ_6 and their physical sense have already been discussed by the authors in [12, 14].

The solutions of system (31), (32), (12)-(14) for initial conditions (18) are given in the form of curves V and VI in Fig. 1-3. A comparison of these curves permits evaluating, all other conditions being equal, the effect of the parameter of three-particle recombination of plasma γ_5 upon its tenfold decrease. As in [14], pronounced extremes in the change of the accelerated mass μ are observed; however, consideration of ionization (γ_6) and recombination (γ_5) does not lead to their noticeable shift during time τ. A tenfold decrease of the parameter γ_5 increases the mass of the accelerated plasma (curve VI, Fig. 1), but, as we see from Fig. 2, this increase does not lead to a noticeable decrease of η and y'.

NOTATION

t	is the time;
N_{el}, N_i	are electron and ion concentration, respectively;
N	is the concentration of third particle participating in the collision;
e	is the electron charge;
T	is the plasma temperature;
z	is the center-of-mass coordinate of plasma;
I	is the current in accelerator;
v	is the center-of-mass velocity of plasma;
m	is the variable accelerated mass of plasma;
V	is the voltage on plates of capacitor bank;
b	is the distributed inductance of unit length of accelerator;
C_0	is the capacitance of capacitor bank;
L_0	is the initial inductance of accelerator with consideration of inductance of capacitor;
R_0	is the ohmic resistance of accelerator;
V_0	is the initial charging voltage of capacitor;
m_0	is the initial mass of plasma formed on breakdown of the gas-discharge gap before the start of its acceleration;
$\alpha_{2recomb}$	is the coefficient of two-particle ion recombination of plasma;
$a_1 - a_6$	are the mass coefficients of diffusion, two-particle recombination, electrode erosion due to ion bombardment of electrodes, electrode erosion due to their Joule melting, triple recombination of electrons and ions, and ionization of neutral gas, respectively;
b_1, b_2, b_3, b_4	are the factors of proportionality taking into account, respectively, the friction of moving plasma on the electrodes (b_1), friction during mass transfer (b_2, b_3), and effect of resistance of external environment (b_4);
τ, y, y', φ, μ, φ'	are the dimensionless variables of time, path, velocity, voltage, mass, and current, respectively;
q, δ_1, δ_2, δ_3, δ_4, γ_1, γ_2, γ_3, γ_4, γ_5, γ_6	are the dimensionless parameters.

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